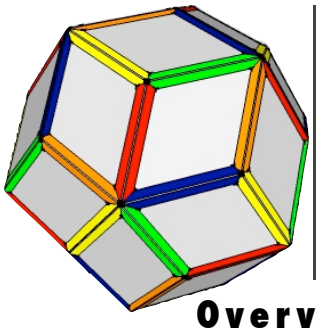
Math 340: Higher Geometry — Fall 2013



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Course Meeting: MWF 8:55am – 10:05am
PPHAC 117

Office Hours: Mon, Tue 1:00-3:00pm, *or by appointment (http://www.doodle.com/klhartshorn)*
PPHAC 215

Overview

Consider the *Common Core Standards* for Mathematics (the national standards for mathematics education that have been adopted by most states, including Pennsylvania), which states:

> An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.
>
> Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

Our purpose in this course is to look at some of the central elements of geometry – questions that have intrigued mathematicians since the days of the ancient Greeks and Chinese. The idea is not to gain a comprehensive background in geometric minutia. This would be impossible in a single semester class. Rather, we will work to find large threads of thought and core questions to be addressed. Stemming from these large ideas, we will address issues of geometric proof, basic concepts, and key ideas that will be useful across the spectrum of geometric study.

For those pursuing certification in education, this course is meant to provide a grounding in *geometric thinking*. Instead of addressing the many specific definitions or theorems in high school mathematics, we will develop tools, techniques, and a frame of mind that should serve in your course development regardless of the particular geometric topics you plan to cover in class.

For the rest of you – whether you are interested in graduate studies in mathematics, a professional career outside of education, or another major besides mathematics – we will be developing a broader sense of our world and how it works. We will explore the interaction between intuition and rigorous proof, and the need for clear definitions and carefully argued proofs. We will develop skills in geometric thinking and visualization, and see how playing with physical models can help develop a deeper understanding of abstract concepts.

Key Ideas for the course

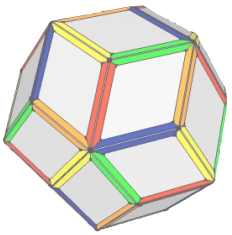
In a single semester, it is impossible to do more than scratch the surface of the range of ideas and topics available in geometry. We will thus focus in this class on the core concepts of geometry: straightness, angle, distance. I hope through this course to help you internalize the following key ideas.

- Most of what is considered *geometry* falls into four main historical categories: art/patterns, navigation/stargazing, machine motion, and building structures.
- The Euclidean geometry we learned in high school is inadequate for understanding many real-world examples.
- The notions of *straightness* and *distance* are central to a geometric understanding our world.
- The most complex of geometric concepts usually boil down to a fundamental understanding of triangles and circles.
- Without precise definitions and careful proofs, our intuition will often lead us to false conclusions.

Course Objectives

To help explore the key ideas of the course, we will work to meet the following outcomes over the course of the semester.

- We will be able to effectively discuss geometric ideas/concepts from various viewpoints (specifically, the art/patterns, navigation/stargazing, machine motion, and building structures strands of geometry).



- We will be able to select and use the appropriate tools and techniques to make geometric arguments and solve geometric problems.
- We will be able to solve problems and construct basic proofs in geometries other than the geometry of the Euclidean plane.
- We will be able to make specific comparisons between various geometries.
- We will be able to make use of fundamental properties of triangles and circles to prove statements in different contexts (i.e.: within alternate geometries).
- When presented with a new problem or concept we will demonstrate the transition from physical experimentation to theoretical proof.

Writing objectives

As a writing-intensive course, we also have several outcomes that are aimed to develop your skill and sophistication as a mathematical writer. Note that whether you plan to teach, pursue higher mathematics, or move on to a career, your skills as a mathematical writer will be of use. A mathematical writer is someone who can write clearly and concisely, who can make use of precise definitions and careful logic to form arguments, who can write proofs that are sufficiently clear for the desired audience. To work toward this ideal . . .

- We will use writing as a means of exploring new geometric ideas and concepts.
- We will use writing and revision as a means of class collaboration to gain a deeper understanding of material in the class.
- We will demonstrate the mastery of mathematical writing conventions in a paper that employs both general discussion and mathematical proof.
- We will write effectively for a range of mathematical audiences: a general audience of non-mathematicians, an audience of mathematics students, and a professional audience.
- We will demonstrate improvement in our writing and thinking about mathematics from a deliberate process of peer-review and revision.

Components of course

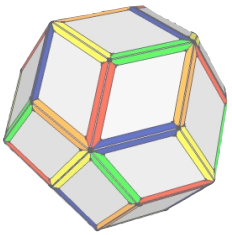
Required materials and texts

The required text for this course is *Experiencing Geometry: Euclidean and Non-Euclidean with History* (3rd Edition), by David Henderson and Daina Taimiņa. ISBN 0-13-143748-8.

The computer program *Geometer's Sketchpad* will be used extensively. Note that while *Sketchpad* is recommended (a 1-year license can be purchased at *http://www.keycurriculum.com/products/sketchpad* for $10), it is available on all campus computers.

In addition, you will need the following materials:

- **Pencil and eraser**. *Do not use pen in this class!* Bring every day to class.
- **Loose-leaf paper**. You need lots of scratch paper for in-class discussion. This can be "recycled" paper from printouts, lined paper that you have purchased, or graph paper. Bring paper every day to class.
- **Spiral notebook, journal, or composition notebook**. You will be keeping a regular diary of your thoughts for this class. Your journal should be dedicated to just this class, and you should have it with you for each class meeting.
- **Ruler** – a 12-inch or 18-inch ruler would be best. Be sure to bring it every day.



Other resources

I recommend bookmarking these sites for use throughout the course. As you prepare for the final research project, you will find some of these sites useful.

- *Euclid's Elements* – this is a complete collection of the 13 volumes of the famous treatise. The propositions include interactive diagrams for illustration.
  http://aleph0.clarku.edu/~djoyce/java/elements/elements.html
- *The Geometry Junkyard* – just as it sounds, this is a collection of random geometric resources, ideas, and inspirations from around the web.
  http://www.ics.uci.edu/~eppstein/junkyard/
- *Geometry Topics at Mathworld* – Mathworld is an on-line encyclopedia of mathematical ideas. Good for basic definitions of new concepts
  http://mathworld.wolfram.com/topics/Geometry.html
- *NonEuclid Java Software* – an on-line app to explore hyperbolic geometry
  http://cs.unm.edu/~joel/NonEuclid/NonEuclid.html

Basic format for class progression

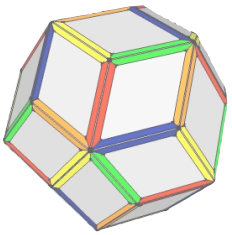
As we work through the semester, the typical flow for learning will be:

- You will read and reflect on some questions at home. Evidence of your reflection will be shown through a personal journal you will keep specifically for the class. Writing in your journal will be checked for completion, but not graded.
- In class, you will work with your team on the questions/problems for the day, aimed at exploring the concepts from the reading more carefully. This exploration will frequently span more than one class, and you will be asked to complete short problems and/or proofs at home as part of your exploration.
- After discussing the topic in class, your team will write a short report on the core questions on the reading.

While this is the ideal cycle of learning, we will regularly interrupt it for some *Sketchpad* activities, the midterm, and key steps of the major research project. In that spirit, the following dates need to be kept in mind.

Key Dates

Friday, September 6	*Sketchpad* lab
Friday, September 27	*Sketchpad* lab
Wednesday, October 9	First midterm due in class
Friday, October 18	*Sketchpad* lab
Monday, October 28	In-class writing workshop
Friday, November 1	*Sketchpad* lab
Monday, November 11	In-class writing workshop
Friday, November 15	*Sketchpad* lab
Friday, November 22	In-class writing workshop
Friday, November 22	Second midterm due in class
Monday, December 2	No class — prepare for presentations and final portfolio
Wednesday, December 4	Class presentations
Friday, December 6	Class presentations
Thursday, December 12	Class presentations (1:30pm-4:30pm)



Grading and Assessment

Your course grade will be computed based on a raw percentage score, broken down as shown in the table below.

10%	Writing and journaling
20%	Discussion and participation
15%	Problem sets (including *Sketchpad* work)
10%	Midterm 1
10%	Midterm 2
35%	Research Project
100%	**Total weight for course**

When computing your score at the end of the semester, an A (+ or –) is typically given to a score of 85% or above, a B (+ or –) to a score between 70% and 85%, a C (+ or –) to a score between 60% and 70%, and a D (+ or –) to a score between 50% and 60%. These values are subject to change and are meant only as a rough guideline, and the final assignment of grades will be determined based on the performance of the entire class and the judgement of the professor.

Writing and Journaling

This course will be focused on "big questions." As such, a lot of learning will be done through journaling and reflection. You will be expected to keep a dedicated journal for this class. Between each class meeting, I will provide a journal-writing assignment. These will be "ungraded," in that your grade will be based on thoughtful responses, not necessarily on accuracy.

Journals will be checked in class (while your team discusses the problems at hand). I will give a check (roughly 80%) for a journal entry that shows a good-faith effort at answering the prompt. A + or – (roughly 100% and 50%, respectively) will be assigned for work that is particularly falls notably outside the expected level of response.

Discussion and Participation

Your grade for discussion and participation will be computed as follows.

10%	Group write-ups
10%	Peer evaluation score
20%	**Total discussion/participation contribution**

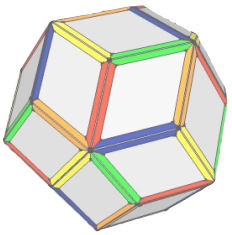
Your group will regularly be asked to write up their conclusions to the problems presented in the text. For each write-up, everyone in the group will receive a common grade for the written solution.

Peer evaluations will be conducted at two points in the semester: just before midterm, and during the last week of class. For the peer evaluation, you will have a certain number of points to distribute among all members of your team (not including yourself). You may not give any two teammates the same number of points, and all teammates must be given a (positive) integer number of points. You are not required to use all the points in your distribution.

For the midterm evaluation, the peer evaluation score will be used to foster a discussion on how to improve cooperation and discussion within the team – the score at this point will be used for midterm evaluations. At the end of the semester, your peer evaluation score (5% of the course grade) will be the sum of the of the points you receive from all of your teammates. This final score will *replace* the score that was shared at midterm.

Problem Sets

Regularly through the semester, you will be given problems to solve at home. All problem sets will be graded not only on mathematical accuracy, but also on quality of writing. You are expected to use complete sentences for all solutions.



As evidence of the revision and work you go through, you will need to submit not only your finalized solution, but also the work that went into writing the solution. For each problem you solve, you will have a neat front page with the clearly written solution. Behind that sheet you will staple the papers that include your scratch work on that problem, including false-starts and preliminary drafts. Note that your grade for the problem will not be affected by the "quality" of your scratch work.

Grading scale

Most graded work in the class will be assessed on a 4 point scale, comparable to the GPA scale, as follows:

- **4 points:** Excellent proof, clearly written with precise language and transparent structure. Outstanding work.
- **3 points:** Acceptable proof, meeting the expectations of the problem. Language may still need some additional precision, or some details of the proof may be elided.
- **2 points:** Solution is missing key steps or contains a significant error. May also indicate writing below the expectations of the class.
- **1 point**: Solution with several significant errors, or completely missing the mark. May indicate complete lack of writing structure (complete sentences, appropriate use of terms, etc.).
- **0 points**: Work was not submitted.

Note that all homework is due by 4:00pm on the date due. Work submitted after 4:00pm will be accepted with a 1 point penalty. Submissions after graded work has been returned to the class will not be accepted.

Exams

There will be two exams in the course — both will be take-home assessments of your progress. See the calendar at the end of this syllabus for the dates of the exams.

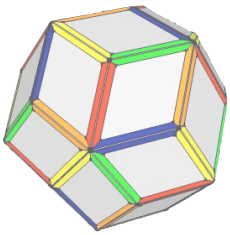
Be sure to mark these dates on your calendar.

Research Project

As the first assignment (initial proposal) approaches, we will discuss the details for the research project. The purpose of the research project is to provide an opportunity to explore and share a topic of geometry outside the scope of the course and to provide an opportunity to practice the writing process in the context of a mathematical project.

The research project will be broken down as follows:

1%	Initial proposal
2%	Prospectus
4%	Progress summaries (2 summaries)
4%	General audience paper
4%	First draft
3%	Referee comments (your comments on classmates' papers)
3%	Revision 1
2%	Class presentation
2%	Final draft and reflection
25%	**Total contribution to course grade**



Course policies and information

Attendance

Attendance is vital for this course. By missing class, you not only detract from your own learning, but you deprive your classmates of the opportunity to learn from your contribution. Team work will be a central part of this course. Your "peer evaluation" score may certainly reflect regular absences.

If you miss more than 3 classes during the semester *for any reason*, your grade will accrue a penalty. On your 4th and each subsequent absence, I will deduct a flat 10% from your class grade.

Attendance is your responsibility. If you miss a class, you will receive a 0 on any quiz or in-class activity that takes place. If your homework is not handed in on time, I will not let you turn it in late because you missed class. Remember, **flight or vacation plans are not acceptable reasons to miss an exam date**. As a general rule, make-up exams are not given. If you have a truly exceptional situation, be sure to see me ***before*** the exam date to discuss your dilemma. If you know that you will be missing a class (due to sports or other activities), let me know ahead of time. Together, we will decide whether alternate arrangements can be made for quizzes or exams or other activities. In all cases, you are responsible for any missed work.

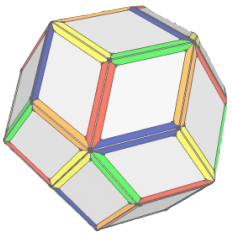
Get to know your classmates! If you know you will be missing a class, have a classmate/roommate/friend bring your homework in for you. As a rule, late work will not be accepted.

Academic Honesty

Students will be expected to adhere to the standard of the Academic Honesty policy as described in the Student Handbook (*http://www.moravian.edu/studentlife/handbook/academic/academic2.html*). Any violations of this will result in severe penalties on the assignment, a report to the Dean, and the very real possibility of failing the course.

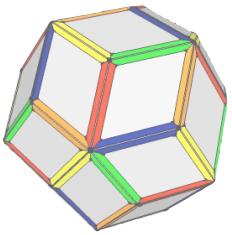
Other reminders, tips, suggestions

- Visit my office: I would love to help address individual issues or answer questions you have about the course or to hear feedback about which aspects of the course are or are not going well. You have a great deal of power to determine the path this class takes -- take advantage of it. You can schedule a time to meet with me using Doodle (*http://www.doodle.com/klhartshorn*).
- You can also communicate with me via e-mail (*hartshorn@math.moravian.edu*).
- This syllabus is subject to change through the semester. An on-line version of the syllabus will be made available during the first week of class and updated as needed.
- If you are in need of special accommodations due to a disability, please contact the Learning Services Office as soon as possible. We can only accommodate your special needs if we are made aware of them.
- Final determination of your course grade is subject to my discretion as professor of the course.

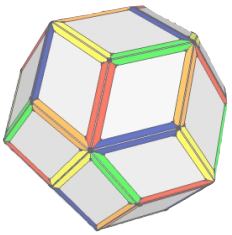


Course Calendar

Aug 26, 2013	Introduction to course. Discuss the notion of "straightness"
Aug 28, 2013	*Reading for class:* Syllabus, Chapters 0– 2 (pages 1–36)
	Journal entry: Class expectations, initial response to 2.1
Aug 30, 2013	*Reading for class:* Chapter 3 from the text.
	Journal entry: Initial responses to 3.1, 3.2
Sep 2, 2013	*Labor Day* — no class
Sep 4, 2013	*Reading for class*: No reading for the day
	Journal entry: Summary of week 1 (what is straight?)
Sep 6, 2013	*Sketchpad Lab*: Meet in PPHAC 112
Sep 9, 2013	*Reading for class*: Chapter 4 (pages 43–51)
	Journal entry: Initial response to 4.1, 4.2
	Research Project: Introduction to assignment
Sep 11, 2013	*Reading for class*: No reading for the day
	Journal entry: Straight lines on a polyhedron
Sep 13, 2013	*Reading for class*: Chapter 5 (pages 59–67)
	Journal entry: Initial response to 5.1
Sep 16, 2013	*Reading for class*: No reading for the day
	Journal entry: The role of models, comparing geometries
Sep 18, 2013	*Reading for class*: Chapter 6 (pages 73–80)
	Journal entry: Initial response to 6.1, 6.2, 6.3
Sep 20, 2013	*Reading for class*: Chapter 6 (pages 80–88)
	Journal entry: Initial response to 6.4 and 6.5
	Research project: Initial topic selection
Sep 23, 2013	*Reading for class*: Chapter 9 (pages 117–121)
	Journal entry: Initial response to 9.1 and 9.2
Sep 25, 2013	*Reading for class*: Chapter 9 (pages 121–124)
	Journal entry: Initial response to 9.3 and 9.4
Sep 27, 2013	*Sketchpad Lab*: Meet in PPHAC 112
Sep 30, 2013	*Reading for class*: Chapter 7 (pages 89–95)
	Journal entry: Initial response to 7.1
Oct 2, 2013	*Reading for class*: Chapter 7 (pages 95–103)
	Journal entry: Initial response to 7.3 and 7.4
Oct 4, 2013	*Reading for class*: Chapter 8 (pages 109–114)
	Journal entry: Initial response to 8.1 and 8.2
	Research project: Prospectus
Oct 7, 2013	*Reading for class*: Chapter 8 (pages 114–116)
	Journal entry: Initial response to 8.3 and 8.4



Oct 9, 2013	*Reading for class*: Chapter 10 (pages 125–129)
	Journal entry: Initial response to 10.1 and 10.2
	First midterm due
Oct 11, 2013	*Reading for class*: Chapter 10 (pages 130–142)
	Journal entry: Initial response to 10.3
Oct 14, 2013	*Fall Break* — no class
Oct 16, 2013	*Reading for class*: No reading for the day
	Journal entry: Summary of work (chapters 1–10)
Oct 18, 2013	*Sketchpad Lab*: Meet in PPHAC 112
Oct 21, 2013	*Reading for class*: Chapter 11 (pages 143–148)
	Journal entry: Initial response to 11.1
Oct 23, 2013	*Reading for class*: Chapter 11 (pages 148–149)
	Journal entry: Initial response to 11.2
Oct 25, 2013	*Reading for class*: No reading for the day
	Journal entry: Progress report on research project
	Research project: First draft of summary article
Oct 28, 2013	*Reading for class*: Read classmate's draft of summary article
	Journal entry: The revision process
	Research project: In-class workshop
Oct 30, 2013	*Reading for class*: Chapter 11 (pages 149–159)
	Journal entry: Initial response to 11.3
Nov 1, 2013	*Sketchpad Lab*: Meet in PPHAC 112
	Journal entry: Response to 11.4
	Research Project: First draft due
Nov 4, 2013	**Research project**: In-class workshop
Nov 6, 2013	*Reading for class*: Chapter 11 (pages 159–164)
	Journal entry: Initial response to 11.5 and 11.6
Nov 8, 2013	*Reading for class*: No reading for the day
	Journal entry: Reflection on patterns and symmetries
Nov 11, 2013	*Reading for class*: Chapter 15 (pages 205–207 and 212–216)
	Journal entry: Initial response to 15.1
Nov 13, 2013	*Reading for class*: Chapter 16 (pages 217–225)
	Journal entry: Initial response to 16.1 and 16.2
Nov 15, 2013	*Reading for class:* Chapter 16 (pages 226–230)
	Journal entry: Progress report on research project
	Sketchpad Lab: Meet in PPHAC 112
Nov 18, 2013	*Reading for class*: No reading for the day
	Journal entry: Apollonius' problem



Nov 20, 2013	*Reading for class*: No reading for the day
	Journal entry: Circle inversions and euclidean geometry
	Research project: Second draft and supplementary materials
Nov 22, 2013	**Research project**: In-class workshop
	Second exam due
Nov 25, 2013	*Thanksgiving break* — no class
Nov 27, 2013	*Thanksgiving break* — no class
Nov 29, 2013	*Thanksgiving break* — no class
Dec 2, 2013	*Preparation for research* — no class
Dec 4, 2013	*Final presentations*
Dec 6, 2013	*Final presentations*
Dec 12, 2013	*Final presentations* 1:30pm-4:30pm
	Research project: Final project portfolios due