# Math 225 - Numerical Analysis Math 392 -Advanced Numerical Analysis <br> Spring 2012 Syllabus 

Class Meetings: Comenius 101<br>Math 225: Tue 8:00-10:05am, Thu 8:00-9:20am<br>Math 392: Tue 8:00-10:05am, Thu 8:45-10:05am<br>Text: An Introduction to Numerical Methods and Analysis (revised edition), by James Epperson<br>Course Website: http://math.moravian.edu/hartshorn/225n392<br>Instructor: Kevin Hartshorn<br>Office Hours: PPHAC 215,<br>Tue 10:15-11:45am, Wed 8:30-10:00am, or by appointment<br>e-mail: hartshorn@math.moravian.edu

## 1 General Comments and Introduction

The purpose of this course is to introduce you to some basic numerical methods for finding approximate solutions to mathematics problems. Finding roots of equations, areas under a curve, best curves to fit given data, etc. are all techniques that are greatly aided with effective computer work.

While almost any computer languages can be employed to tackle the problems of this course, we will focus on using Maple for solving these problems. There will be some programming required in this course, but the focus is using the computer algorithms and scripts to gain a better understanding and appreciation of how to tackle real mathematical problems from a computational viewpoint.

## 2 Goals and Objectives

- Be able to select and apply basic numerical methods for solving problems.
- Be able to perform simple programming in Maple, including writing procedures, importing/exporting data, and working with various data types.
- Be able to analyze numerical techniques for simple problems, discussing issues of accuracy, efficiency, and stability.


## 3 Grading and Assessment

The components of the course will be weighted as shown in the table below.

| $10 \%$ | Reading responses |
| :--- | :--- |
| $25 \%$ | Homework submissions (12 submissions) |
| $20 \%$ | Larger-scaled projects (average over all projects) |
| $25 \%$ | Exams (2 exams) |
| $20 \%$ | Final Exam (Friday, May 4 at 6:30pm) |
| $100 \%$ | Total |

Generally speaking, your final course grade translates to a letter grade loosely based on the standard 4-point system: generally $85 \%$ marks the difference between an "A" and a "B", $70 \%$ marks the difference between a "B" and a "C", $60 \%$ marks the difference between a "C" and a "D", and any score below $50 \%$ is
considered failing. Note that these numbers are meant only as a guideline and are subject to change over the course of the semester. See the Student Handbook ${ }^{1}$ for a qualitative description for the various grades.

## 4 Course Format

Every day of class, you will be expected to have your book, as well as paper and pencil for taking notes and working through exercises. I will provide frequent opportunities to work through ideas in the book with your classmates.

### 4.1 Reading Responses

For each class, you will be expected to read the relevant section of the book before we discuss it together. See the calendar at the end of this syllabus for the schedule of readings. I expect that each section will require at least 30 to 60 minutes to read through carefully. Be sure to take notes, try to follow through the examples yourself, and be willing to go back to earlier pages to review material.

For each reading, you will be asked to submit a short reflection on the reading. You should aim for less than 100 words, and submit your response by e-mail before midnight on the day we discuss the reading. You will receive credit for any response that reflects that you have carefully read the relevant sections. In writing your response, you might consider addressing some of the following questions. You can also use these questions to assess how well you have worked through the readings.

- What are the key techniques or ideas introduced in this reading?
- What are some problems that you should be able to solve using this material? In particular, look to problems from the corresponding exercise set.
- How does the current reading connect to ideas/concepts/techniques that you already know?
- What new notation, vocabulary, or formulas are needed to fully understand this section?

Keep in mind that the point is to provide a short feedback to let me know (1) that you have read through the section and (2) what you really want to talk about when we meet in class.

## Thursday class meetings

Frequently, 225 and 392 will have the same required readings. In our Thursday meetings, I may spend more time with the 225 students discussing the reading itself - strategies for reading, parsing the sentences, working through the text. I will expect the 392 students to do these things on their own, and will spend more time in the Thursday sessions discussing the implications of the readings, with the assumption that you have more thoroughly worked through the text on your own.

You are welcome and encouraged to stop by my office to discuss upcoming readings both before and after they are discussed in the class.

### 4.2 Homework problems

There will by 13 homework assignments through the semester, but I will drop the lowest score and compute your homework grade as an average of the 12 remaining scores. See the class schedule at the end of this syllabus for the list of homework assignments.

Homework is expected to be neat, organized, and clearly show all work. You are more than welcome to type your response, though a (neatly) hand-written response is also acceptable. All solutions should clearly indicate both the problem and the logical structure of the solution. Sloppy homework will be penalized.

In completing your homework, feel free to use:

[^0]- Your text, class notes, or any material from the course web page.
- Any capabilities of Maple (except when specifically told otherwise).
- The Maple support page (http://www.maplesoft.com/support/help/).
- Consultation with the professor.
- Consultation with your classmates. Note that each person should write his or her own solutions. When asked to test procedures with your own data or functions, each person should include their own inputs.
You may not simply copy identical answers from each other's homework. You may not consult people or resources outside of class in completing your homework assignments.


## Electronic or physical submissions

Any Maple code called for in a homework assignment must be submitted electronically. All other work may be submitted either electronically or in hard copy.

- Electronic submissions: The class web page http://math.moravian.edu/hartshorn/225n392/ has a form for submitting work electronically. The file name for your work will automatically be tagged with your student ID. You may submit multiple files for a given homework assignment. Your submission must be made by 4 pm on the due date or it will be considered late.
- Hard copy submissions: You may submit your work to me in class or leave it in the box outside my office (PPHAC 215). All work must be submitted by 4 pm on the due date or it will be considered late.


## Late work

I will accept work that is up to 24 hours late - that is, by 4 pm the day after the due date. Such work will be accrued a penalty.

After that time, I will be posting solutions to the homework sets and will no longer accept late work.

## Assessment of work

I will typically grade each homework problem on a 3 -point scale. Your homework grade will be recorded as a percentage score, based on the average of the homework problems for that assignment.
$300-$ level students will be expected to use complete sentences and careful mathematical argument in all submitted problems. Assessment for 300 -level submissions will be based not just on finding the right answer, but demonstrating clear and precise mathematical thinking.

### 4.3 Applied Projects

There will be several applied projects that you will complete in groups. For the each project submission, there will be a group score that you will then distribute among the group members to reflect the level of contribution that each person has made to the project.

The topics for the projects will be determined over the course of the semester. Some possible topics include: maneuvering a robotic arm, drawing with splines, creating images using PDF, or GPS navigation.

### 4.4 Exams

There will be two exams. Both exams will be closed-book, in-class exams. The dates of the exams are Thursday, February 16 and Thursday, April 5. For each exam, you will begin the test at your regular meeting time ( 225 begins at 8:00, 392 begins at 8:45). Please be respectful of fellow test-takers as you enter and leave the room.

The scheduled time for the final exam is Friday, May 4 at $6: 30 \mathrm{pm}$. The format for the final exam will be discussed after the Spring break.

## 5 Attendance and other Issues

### 5.1 Technology

Maple is required software for this course. Many homework problems will require the use of Maple, most algorithms will be discussed using the Maple programming language, and sample scripts and procedures will use the Maple platform. Procedures and sample files will be posted on the class web page (http://math.moravian.edu/hartshorn/225n392/). Any material posted on the site may be used in completing homework.

While all computers on the campus network have a version of Maple installed, you can purchase a student license for Maple at https://webstore.maplesoft.com/catalog.aspx.

Those familiar with Python may opt to experiment with the software Sage ${ }^{2}$, which is a Python-based freeware alternative to Maple. I will accept solutions sets using either platform. Note that Sage is not installed on campus computers, but may be freely downloaded to install on your own machine.

## 200 versus 300 level in a single course

This course is being offered at both the 200 and 300 level. This is not a difference that translates to "300level students have to do more stuff." Rather, it indicates a higher set of expectations for the 300-level students reflecting deeper and more independent thinking. A few points of comparison between 200-level and 300-level courses:

1. At the 200-level, students are just beginning to write their own proofs, often with assistance from the professor or step-by-step suggestions in the text. Proofs are shorter and require fewer logical steps. At the 300-level students are expected to develop their own proofs, including proofs that require several logical steps or indirect methods.
2. At the 200-level, students may be expected to learn material on their own (e.g. reading from the text), but readings are often shorter and significant scaffolding/assistance is provided by the professor. At the 300-level, student have a greater expectation to learn material on their own and spend more time in class applying the reading to problem-solving and deeper connections.
3. While proofs are introduced to students at the 200 -level, more complex proofs may only be provided as a "proof sketch" or left out of the class discussion. At the 300-level more results are expected to be proven rigorously within the course.
4. Problem-solving at the 200-level, while more complex than that at the 100 -level does not require the same level of independent thinking and/or creativity that 300 -level problem sets might require.

In this course, these differences will translate as follows:

- The number of problems assigned will be similar (perhaps not exactly the same). I will not say that all the 200 students have to do problem $x$ while 300 students have to do $x$ and $y$. I will instead ask 200 students to work on problems $x$ and $y$ while the 300 students work on $\alpha$ and $\beta$.
- Problems at the 300-level will be more likely to require some degree of proof or mathematical argument that is not generally expected at the 200-level. Proof and arguments from Math 216 (Discrete Structures and Proof) will be assumed knowledge for students at the 300 -level. I will also have higher expectations for the quality of responses from the 300-level students on both homework and tests.
- Everyone will be expected to read the text independently. However, students at the 200-level can expect more in-class scaffolding and assistance with the reading. Students at the 300 -level will have a greater expectation to complete the readings on their own (although I am more than happy to help with the readings in office hours).

[^1]
### 5.2 Attendance

While I expect you to attend every session, unavoidable situations will arise during the semester. This course does not have an official attendance policy. However, keep in mind the following:

- The due date for homework assignments is independent of your attendance: the homework policy for late homework still applies, regardless the reason for your absence. If you know that you will be missing a class, it is your responsibility to get the homework to me.
- If you know that you will miss a class that requires your presence (e.g.: one of the midterms), let me know as soon as possible so that other arrangements can be made.
- While any Maple worksheets will often be posted on the class web page, it is your responsibility to get class notes from either myself or from your classmate if you miss class.


### 5.3 Academic Honesty

Everyone is expected to adhere to Moravian College's Academic Honesty policy, as described in the Student Handbook ${ }^{3}$.

### 5.4 Office Hours

Visit my office! You have a great deal of power to determine the path this class takes - take advantage of it. Do not wait until the midterm to visit my office. Some reasons to stop by are:

- Discuss the reading for class the next day -this gives you a chance to make sure your questions or concerns are addressed so that you will feel more engaged in the class.
- Discuss material that was covered in class. Work we do in the class will be tested on the midterm. By coming to office hours, you can make sure you understand the material or help me see that we really need to discuss a particular topic more carefully.
- Discuss problems with the homework. This will help ensure that you do as well as possible on the homework. Note that homework problems are not just a repeat of what was done in class, but are an active part of the learning process.
- Discuss general questions/comments about the course. Is it going to fast? Too slow? Do you feel that we are meeting the course objectives? Use office hours as a way of making sure that the class serves you in the best way possible.

If you can't stop by my office hours, send me e-mail (hartshorn@math.moravian.edu) to set an alternative time to meet.

### 5.5 Final Disclaimers

This syllabus is subject to change through the semester. The most recent version of the syllabus will be posted at http://www.math.moravian.edu/hartshorn/225n392/.
Final determination of your course grade is subject to my discretion as professor of the course.

[^2]
## Planned Schedule for Spring 2012

| Date | Topic and readings |  | Homework due |
| :---: | :---: | :---: | :---: |
| 1/17 | 225/392 | §1.1: Basic tools of calculus | Purchase book and bring it to class |
| 1/19 | 225/392 | §1.2: Error, Approximate Equality, and Asymptotic Order Notation <br> §1.3: A Primer on Computer Arithmetic | Comments on syllabus |
| 1/24 | 225/392 | §1.5: Simple Approximations <br> §1.6: Approximating the Natural Logarithm | Calculus worksheet |
| 1/26 | Math 225 <br> Math 392 | §2.1: Horner's Rule and Nested Multiplication <br> §2.2: Difference Approximations to the Derivative | $\begin{aligned} & \S 1.2: 5,8,12 \\ & \S 1.3: 3(\mathrm{a}, \mathrm{c}, \mathrm{~d}), 7,10 \\ & \hline \S 1.2: 8,12,13 \\ & \S 1.3: 3(\mathrm{a}, \mathrm{c}, \mathrm{~d}), 7,11 \end{aligned}$ |
| 1/31 | 225/392 | §2.4: Linear Interpolation <br> §2.5: The Trapezoid Rule | - |
| 2/2 | 225/392 | §2.6: Solutions of Tridiagonal Linear Systems | $\begin{aligned} & \text { §1.5: } 10,11 \\ & \S 1.6: 9 \\ & \S 2.1: 4 \\ & \S 2.2: 8,9(\mathrm{~b}, \mathrm{~d}) \end{aligned}$ |
| 2/7 | 225/392 | §3.1: The Bisection Method <br> §3.2: Newton's Method: Derivation and Examples | - |
| 2/9 | Math 225 | §3.3: How to Stop Newton's Method <br> §3.4: Division Using Newton's Method <br> §3.5: The Newton Error Formula | $\begin{aligned} & \S 2.4: 10 \\ & \S 2.5: 14 \\ & \S 2.6: 4,5,6,11 \end{aligned}$ |
|  | Math 392 | §3.3: How to Stop Newton's Method <br> §3.5: The Newton Error Formula <br> $\S 3.6$ : Theory and Convergence | $\begin{aligned} & \S 2.4: 10 \\ & \S 2.5: 17 \\ & \S 2.6: 4,5,6,12 \end{aligned}$ |
| 2/14 | Math 225 | §3.7: Computation of the Square Root | $\begin{aligned} & \S 3.1: 3(\mathrm{a}, \mathrm{e}, \mathrm{~h}), 9,14 \\ & \S 3.2: 7,8 \\ & \S 3.3: 3(\mathrm{~b}, \mathrm{e}, \mathrm{~h}), 6 \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \S 3.1: 3(\mathrm{a}, \mathrm{e}, \mathrm{~h}), 9,14 \\ & \S 3.2: 7,8 \\ & \S 3.3: 3(\mathrm{~b}, \mathrm{e}, \mathrm{~h}), 2 \end{aligned}$ |
| 2/16 | Math 225 <br> Math 392 | Midterm 1 - each group starts test at regular meeting |  |
| 2/21 | 225/392 | §3.8: The Secant Method: Derivations and Examples <br> §3.9: Fixed Point Iteration | - |
| 2/23 | Math 225 | §3.10.1: Extrapolation and Acceleration <br> $\S 3.10 .2$ : Variants of Newton's Method <br> §3.10.5: Hybrid Algorithms | $\begin{aligned} & \S 3.4: 4 \\ & \S 3.5: 1,2,9 \\ & \S 3.7: 1,10 \end{aligned}$ |


| Date |  | Topic and readings | Homework due |
| :---: | :---: | :---: | :---: |
|  | Math 392 | §3.10.1: Extrapolation and Acceleration <br> §3.10.3: Secant Method Theory and Convergence <br> §3.10.4: Multiple Roots | $\begin{aligned} & \text { §3.5: } 9 \\ & \S 3.6: 6,8,9 \\ & \S 3.7: 1,5 \\ & \hline \end{aligned}$ |
| 2/28 | 225/392 | §4.1: Lagrange Interpolation <br> §4.2: Newton Interpolation and Divided Differences | - |
| 3/1 | Math 225 <br> Math 392 | §4.3: Interpolation Error <br> §4.4: Muller's Method and Inverse Quadratic Interpolation | $\begin{aligned} & \S 3.8: 4,13 \\ & \S 3.9: 5(\mathrm{~b}, \mathrm{~d}, \mathrm{e}), 7,12 \\ & \S 3.10: 3,6(\mathrm{a}, \mathrm{e} \mathrm{~h}), 8(\mathrm{a}, \mathrm{e}, \mathrm{~h}), 23 \\ & \hline \S 3.8: 3,4,13 \\ & \S 3.9: 5(\mathrm{~b}, \mathrm{~d}, \mathrm{e}), 11,12 \\ & \S 3.10: 3,15,17,20 \end{aligned}$ |
| $\begin{aligned} & \hline 3 / 6 \\ & 3 / 8 \end{aligned}$ |  | Spring Break |  |
| 3/13 | 225/392 | §4.7: Piecewise Polynomial Interpolation <br> §4.8: An Introduction to Splines | - |
| 3/15 | Math 225 | §4.10: An Introduction to Splines | $\begin{aligned} & \hline \S 4.1: \\ & \S 4.2: \\ & \S 4.3: \\ & \S 4.4 \end{aligned}$ |
|  | Math 392 | §4.10: An Introduction to Splines <br> §4.11.1: Stability of Polynomial Interpolation <br> §4.11.3: The Chebyshev Nodes | $\begin{aligned} & \hline \S 4.1: \\ & \S 4.2: \\ & \S 4.3: \\ & \S 4.4 \\ & \hline \end{aligned}$ |
| 3/20 | 225/392 | §5.1: A Review of the Definite Integral <br> §5.2: Improving the Trapezoid Rule <br> §5.4: The Midpoint Rule | - |
| 3/22 | Math 225 <br> Math 392 | §5.3: Simpson's Rule and Degree of Precision <br> §5.5: Stirling's Formula |  |
| 3/27 | Math 225 | §5.7: Extrapolation Methods | $\begin{aligned} & \S 5.1: \\ & \S 5.2: \\ & \S 5.3: \\ & \S 5.4: \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \S 5.1: \\ & \S 5.2: \\ & \S 5.3: \\ & \S 5.4: \\ & \hline \end{aligned}$ |
| 3/29 | Math 225 | Midterm 2 - each group starts test at regular meet | me |


| Date | Topic and readings |  | Homework due |
| :---: | :---: | :---: | :---: |
|  | Math 392 |  |  |
| $4 / 3$ | 225/392 | §5.8.1: Romberg Integration <br> §5.8.3: Adaptive Integration <br> §5.8.4: Peano Estimates for the Trapezoid Rule | - |
| $4 / 5$ | Math 225 | §7.1: Linear Algebra Review <br> §7.2: Linear Systems and Gaussian Elimination | $\begin{aligned} & \S 5.5: \\ & \S 5.7: \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \S 5.5: \\ & \S 5.7: \\ & \hline \end{aligned}$ |
| 4/10 | 225/392 | §7.3: Operation Counts <br> §7.4: The $L U$ Factorization | - |
| 4/12 | Math 225 | §7.5: Perturbation, Conditioning, and Stability | $\begin{aligned} & \S 5.8: \\ & \S 7.1: \\ & \S 7.2: \\ & \hline \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \hline \S 5.8: \\ & \S 7.1: \\ & \S 7.2: \end{aligned}$ |
| 4/17 | 225/392 | §7.6: SPD Matrices and the Cholesky Decomposition | - |
| 4/19 | Math 225 | §7.7: Iterative Methods for Linear Systems | $\begin{aligned} & \S 7.3: \\ & \S 7.4: \\ & \S 7.5: \\ & \hline \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \hline \S 7.3: \\ & \S 7.4: \\ & \S 7.5: \\ & \hline \end{aligned}$ |
| $4 / 24$ | 225/392 | §7.8: Nonlinear Systems: Newton's Method and Related Ideas | - |
| 4/26 | Math 225 | Closing comments and discussion | $\begin{aligned} & \hline \S 7.6: \\ & \S 7.7: \\ & \S 7.8: \\ & \hline \end{aligned}$ |
|  | Math 392 |  | $\begin{aligned} & \S 7.6: \\ & \S 7.7: \\ & \S 7.8: \\ & \hline \end{aligned}$ |
| 5/4 | Friday | Final Exam - 6:30pm |  |


[^0]:    ${ }^{1}$ http://www.moravian.edu/studentLife/handbook/academic/academic.html

[^1]:    ${ }^{2}$ http://www.sagemath.org/

[^2]:    ${ }^{3}$ http://www.moravian.edu/studentLife/handbook/academic/academic2.html

