# Math 340: Higher Geometry

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Course Meeting:	MWF 7:30am – 8:40am PPHAC 235
Office Hours:	Tuesdays 8:00-10:00am and 1:00-3:00pm <i>, or by appointment</i> PPHAC 215

# **Overview**

Consider the *Common Core Standards* for Mathematics (the national standards for mathematics education that have been adopted by most states, including Pennsylvania), which states:

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

Our purpose in this course is to look at some of the central elements of geometry – questions that have intrigued mathematicians since the days of the ancient Greeks and Chinese. The idea is not to gain a comprehensive background in geometric minutia. This would be impossible in a single semester class. Rather, we will work to find large threads of thought and core questions to be addressed. Stemming from these large ideas, we will address issues of geometric proof, basic concepts, and key ideas that will be useful across the spectrum of geometric study.

For those pursuing certification in education, this course is meant to provide a grounding in *geometric thinking*. Instead of addressing the many specific definitions or theorems in high school mathematics, we will develop tools, techniques, and a frame of mind that should serve in your course development regardless of the particular geometric topics you plan to cover in class.

For the rest of you – whether you are interested in graduate studies in mathematics, a professional career outside of education, or another major besides mathematics – we will be developing a broader sense of our world. We will explore the interaction between intuition and rigorous proof, and the need for clear definitions and carefully argued proofs. We will develop skills in geometric thinking and visualization, and see how playing with physical models can help develop a deeper understanding of abstract concepts.

# Key Ideas for the course

In a single semester, it is impossible to do more than scratch the surface of the range of ideas and topics available in geometry. We will thus focus in this class on the core concepts of geometry: straightness, angle, distance. I hope through this course to help you internalize the following key ideas.

- Most of what is considered *geometry* falls into four main historical categories: art/patterns, navigation/stargazing, machine motion, and building structures.
- The Euclidean geometry we learned in high school is inadequate for understanding many real-world examples.
- The notions of straightness and distance are subject to change and central to understanding our world.
- The most complex of geometric concepts usually boil down to a fundamental understanding of triangles and circles.
- Without precise definitions and careful proofs, our intuition will lead us to false conclusions.

## **Course Objectives**

To help explore the key ideas of the course, we will work to meet the following outcomes over the course of the semester.

• We will be able to effectively discuss geometric ideas/concepts from various viewpoints (specifically, the art/patterns, navigation/stargazing, machine motion, and building structures strands of geometry).

- We will be able to select and use the appropriate tools and techniques to make geometric arguments and solve geometric problems.
  - We will be able to solve problems and construct basic proofs in geometries other than the geometry of the Euclidean plane.
- We will be able to make specific comparisons between various geometries.
- We will be able to make use of fundamental properties of triangles and circles to prove statements in different contexts (i.e.: within alternate geometries).
- When presented with a new problem or concept we will demonstrate the transition from physical experimentation to theoretical proof.

## Writing objectives

As a writing-intensive course, we also have several outcomes that are aimed to develop your skill and sophistication as a mathematical writer. Note that whether you plan to teach, pursue higher mathematics, or move on to a career, your skills as a mathematical writer will be of use. A mathematical writer is someone who can write clearly and concisely, who can make use of precise definitions and careful logic to form arguments, who can write proofs that are sufficiently clear for the desired audience. To work toward this ideal . . .

- We will use writing as a means of exploring new geometric ideas and concepts.
- We will use writing and revision as a means of class collaboration to gain a deeper understanding of material in the class.
- We will demonstrate the mastery of mathematical writing conventions in a paper that employs both general discussion and mathematical proof.
- We will write effectively for a range of mathematical audiences: a general audience of non-mathematicians, an audience of mathematics students, and a professional audience.
- We will demonstrate improvement in our writing and thinking about mathematics from a deliberate process of peer-review and revision.

# **Components of course**

## **Required materials and texts**

The required text for this course is *Experiencing Geometry: Euclidean and Non-Euclidean with History* (3<sup>rd</sup> Edition), by David Henderson and Daina Taimiņa. ISBN 0-13-143748-8.

The computer program *Geometer's Sketchpad* will be used extensively. Note that while *Sketchpad* is recommended (and can be purchased at *http://www.keypress.com/x24119.xml* for less than \$30), it is available on all campus computers.

In addition, you will need the following materials:

- Pencil and eraser. Do not use pen in this class! Bring every day to class.
- Loose-leaf paper. You need lots of scratch paper for in-class discussion. This can be "recycled" paper from printouts, lined paper that you have purchased, or graph paper. Bring paper every day to class.
- Spirial notebook, journal, or composition notebook. You will be keeping a regular diary of your thoughts for this class. Your journal should be dedicated to just this class, and you should have it with you for each class meeting.
- Ruler a 12-inch or 18-inch ruler would be best. Be sure to bring it every day.
- Crochet hook- size H, I, or J. This can be found at Michael's, Wal-Mart, AC Moore, or any store that sells craft supplies. You need to have purchased this hook to bring to class on Monday, September 12.



Other resources

I recommend bookmarking these sites for use throughout the course. As you prepare for the final research project, you will find some of these sites useful.

• Ball of yarn – along with the crochet hook, this needs to be purchased by September 12. A basic (cheap) acrylic

- Euclid's Elements this is a complete collection of the 13 volumes of the famous treatise. The propositions include interactive diagrams for illustration. http://aleph0.clarku.edu/~djoyce/java/elements/elements.html
- The Geometry Junkyard just as it sounds, this is a collection of random geometric resources, ideas, and inspirations from around the web. http://www.ics.uci.edu/~eppstein/junkyard/
- *Geometry Topics at Mathworld* Mathworld is an on-line encyclopedia of mathematical ideas. Good for basic definitions of new concepts http://mathworld.wolfram.com/topics/Geometry.html
- NonEuclid Java Software an on-line app to explore hyperbolic geometry http://cs.unm.edu/~joel/NonEuclid/NonEuclid.html

## Basic format for class progression

yarn is sufficient (e.g.: Red Heart brand).

As we work through the semester, the typical flow for learning will be:

- You will read and reflect on some questions at home. Evidence of your reflection will be shown through a personal journal you will keep specifically for the class. Writing in your journal will be checked for completion, but not graded.
- In class, we will have short assessments of your reading. Sometimes these will be done as individuals, and sometimes you will work as a team to complete the assessment.
- After the (short) assessment, you will work with your team on the questions/problems for the day, aimed at
  exploring the concepts from the reading more carefully. This exploration will frequently span more than one class,
  and you will be asked to complete short problems and/or proofs at home as part of your exploration.
- After having discussed the topic in class, you will review your thoughts on the reading and write a short report on the core questions on the reading. At this time, you will also reflect on how your thinking has evolved.

While this is the ideal cycle of learning, we will regularly interrupt it for some *Sketchpad* activities, the midterm, and key steps of the major research project. A preliminary course calendar will be distributed as a separate handout, but the following dates need to be kept in mind.

Friday, September 9:	Sketchpad lab
Wednesday, October 5:	In-class midterm
Friday, October 21:	Sketchpad lab
Friday, October 28:	Workshopping session on research project
Friday, November 4:	Sketchpad lab
Friday, November 18:	Sketchpad lab
Monday, November 21:	Workshopping session on research project
Fri., Dec. 2 – Wed., Dec. 7:	Research project presentations
Thu., December 15, 8:30am:	Final exam

# **Grading and Assessment**

Your course grade will be computed based on a raw percentage score, broken down as shown in the table below.



- 15% Writing and journaling
- 15% Discussion and participation
- 15% Problem sets (including Sketchpad work)
- 15% Midterm
- 20% Research Project
- 20% Final Exam

#### 100% Total weight for course

When computing your score at the end of the semester, an A (+ or –) is typically given to a score of 85% or above, a B (+ or –) to a score between 70% and 85%, a C (+ or –) to a score between 60% and 70%, and a D (+ or –) to a score between 50% and 60%. These values are subject to change and are meant only as a rough guideline, and the final assignment of grades will be determined based on the performance of the entire class and the judgement of the professor.

## Writing and Journaling

This course will be focused on "big questions." As such, a lot of learning will be done through journaling and reflection. You will be expected to keep a dedicated journal for this class. Between each class meeting, I will provide a journal-writing assignment. These will be "ungraded," in that your grade will be based on thoughtful responses, not necessarily on accuracy.

In addition to the regular journal entries, you are welcome and encouraged to share your daily thoughts about the course and the subject in your journals. Note that anything that you write in the journal will only be read by you and me.

I will collect journals roughly once per week.

## **Discussion and Participation**

Much of the class will be spent working with your teams. For most new readings, class will begin with a short Readiness Assessment Test (RAT). Sometimes the RAT will be done individually (iRAT), sometimes it will be as a team (tRAT), and sometimes it will be both (individually first, then as a team). When both individual and team RATs are given for a reading, the average of the two scores will be entered for your RAT grade.

Your grade for discussion and participation will be computed as follows.

- 10% Average of RAT scores
- 5% Peer evaluation score

#### 15% Total discussion/participation contribution

Peer evaluations will be conducted at two points in the semester: just before midterm, and during the last week of class. For the peer evaluation, you will have a certain number of points to distribute among all members of your team (not including yourself). You may not give any two teammates the same number of points, and all teammates must be given a (positive) integer number of points. You are not required to use all the points in your distribution.

For the midterm evaluation, the peer evaluation score will be used to foster a discussion on how to improve cooperation and discussion within the team – the score at this point will be used for midterm evaluations. At the end of the semester, your peer evaluation score (5% of the course grade) will be the sum of the of the points you receive from all of your teammates. This final score will *replace* the score that was shared at midterm.

## **Problem Sets**

Regularly through the semester, you will be given problems to solve at home. All problem sets will be graded not only on mathematical accuracy, but also on quality of writing. You are expected to use complete sentences for all solutions.



As evidence of the revision and work you go through, you will need to submit not only your finalized solution, but also the work that went into writing the solution. For each problem you solve, you will have a neat front page with the clearly written solution. Behind that sheet you will staple the papers that include your scratch work on that problem, including false-starts and preliminary drafts. Note that your grade for the problem will not be affected by the "quality" of your scratch work.

Each problem will be assessed on a 10 point scale, distributed as follows:

- 4 points: Is the proof or problem complete and mathematically correct? Partial credit is likely.
- 4 points: Is your solution neat and well-written? Partial credit is likely.
- 2 points: Did you include your preliminary work on the problem? No partial credit either you did or did not.

Note that all homework is due by 4:00pm on the date due. Work submitted after 4:00pm will be accepted with a 30% penalty. Submissions after graded work has been returned to the class will not be accepted.

#### Exams

There will be one midterm and a final exam in the course. The date for the midterm is Wednesday, October 12, 2011. We will work out the details for the midterm together as the date approaches. The final exam will be on Thursday, December 15, 2011 at 8:30 AM. The final exam takes place in our regular meeting room.

Be sure to mark these dates on your calendar. Remember, **flight or vacation plans are not acceptable reasons to miss an exam date**. As a general rule, make-up exams are not given. If you have a truly exceptional situation, be sure to see me *before* the exam date to discuss your dilemma.

## **Research Project**

As the first assignment (initial proposal) approaches, we will discuss the details on assessing the research project. However, thoughtful completion of all the requirements in a timely manner will result in a passing grade (C or B). A higher grade is garnered through excellent work. The purpose of the research project is to provide an opportunity to explore and share a topic of geometry outside the scope of the course and to provide an opportunity to practice the writing process in the context of a mathematical project.

The grade for the project will be broken down as follows:

- 1% Initial proposal
- 2% Prospectus
- 4% Progress summaries (2 summaries)
- 4% General audience paper
- 4% First draft
- 3% Referee comments (your comments on classmates' papers)
- 3% Revision 1
- 2% Class presentation
- 2% Final draft and reflection
- 25% Total contribution to course grade

# Course policies and information

#### Attendance

Attendance is vital for this course. By missing class, you not only detract from your own learning, but you deprive your classmates of the opportunity to learn from your contribution. Team work will be a central part of this course. Your "peer evaluation" score may certainly reflect regular absences.



If you miss more than 3 classes during the semester *for any reason*, your grade will accrue a penalty. On your 4<sup>th</sup> and each subsequent absence, I will deduct a flat 10% from your class grade.

Attendance is your responsibility. If you miss a class, you will receive a 0 on any quiz or in-class activity that takes place. If your homework is not handed in on time, I will not let you turn it in late because you missed class. If you know that you will be missing a class (due to sports or other activities), let me know ahead of time. Together, we will decide whether alternate arrangements can be made for quizzes or exams or other activities. In all cases, you are responsible for any missed work.

**Get to know your classmates!** If you know you will be missing a class, have a classmate/roommate/friend bring your homework in for you. As a rule, late work will not be accepted.

## Academic Honesty

Students will be expected to adhere to the standard of the Academic Honesty policy as described in the Student Handbook (*http://www.moravian.edu/studentlife/handbook/academic/academic2.html*). Any violations of this will result in severe penalties on the assignment, a report to the Dean, and the very real possibility of failing the course.

## Other reminders, tips, suggestions

- Visit my office: I would love to help address individual issues or answer questions you have about the course or to hear feedback about which aspects of the course are or are not going well. You have a great deal of power to determine the path this class takes -- take advantage of it. You can also communicate with me via e-mail (hartshorn@math.moravian.edu).
- Take advantage of the tutoring center. Beginning around the second week of class, the math tutoring center is open Monday through Thursday evenings in the Math/CS reading room (PPHAC 238).
- This syllabus is subject to change through the semester. The most recent version of the syllabus can be found at *http://www.math.moravian.edu/hartshorn/math340/.*
- If you are in need of special accommodations due to a disability, please contact the Learning Services Office as soon as possible. We can only accommodate your special needs if we are made aware of them.
- Final determination of your course grade is subject to my discretion as professor of the course.